Plan of the lectures:

- Intro + overview
- CR sources and acceleration
- Galactic cosmic rays
- Extragalactic cosmic rays
- Photons
- Neutrinos
- Gravitational waves

Units and distances

- Energies: $1\,\mathrm{erg} = 10^{-7}\mathrm{J} \simeq 624\,\mathrm{GeV}$

  $\hbar = c = 1$: $[E] = [p] = [m] = \mathrm{GeV}$

  GeV, TeV, PeV, EeV

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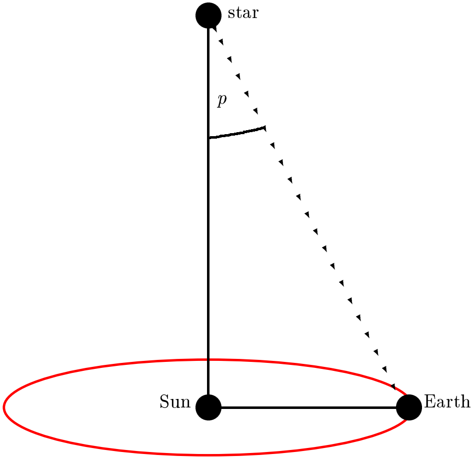
- energy of a CR nuclei with mass number A:
  - total energy E: at HE, A is difficult to measure
  - energy/nucleon E/A: conserved in spallation reactions $A \to A_1 + A_2$

    $$E_1 = E/A_1 \qquad \text{and} \qquad E_2 = E/A_2$$

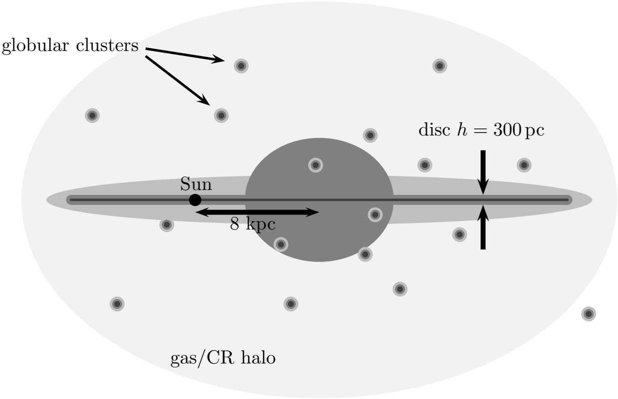
  - rigidity $\mathcal{R} = \frac{cp}{Ze}$: CRs with same $\mathcal{R}$ follows same trajectory in $\boldsymbol{B}$

Units and distances

Measuring parallaxes: $1\mathrm{pc} = 3.08 \times 10^{18}\,\mathrm{cm}$



Milky Way



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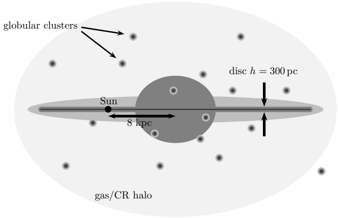
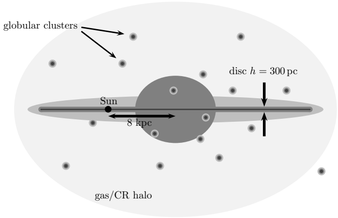


- Larmor radius

$1\,\mathrm{pc} = 3.1 \times 10^{18}$ cm

$$R_L = \frac{cp}{ZeB} = \frac{\mathcal{R}}{B} \simeq 1.08\,\mathrm{pc}\,\frac{\mu\mathrm{G}}{B}\,\frac{E}{Z \times \mathrm{PeV}}$$

Milky Way



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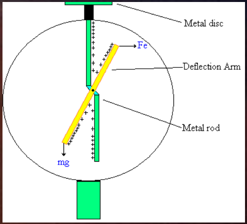
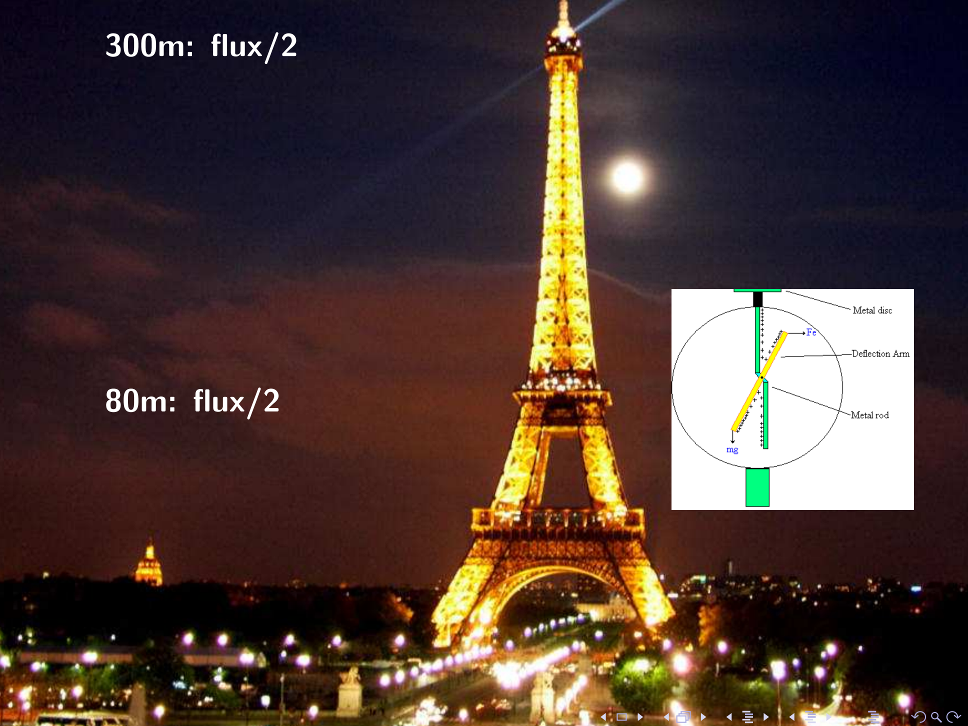
- extragalactic scales:
  - distance to Virgo: 18 Mpc
  - observable universe: $c/H_0 \sim 4$ Gpc

1910: Father Wulf measures ionizing radiation in Paris

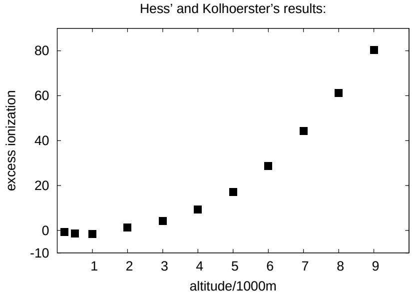
80m: flux/2



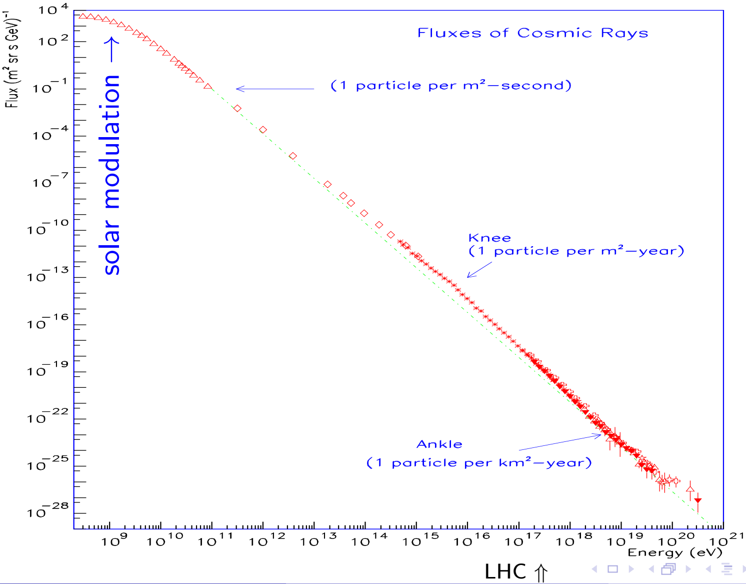

300m: flux/2
80m: flux/2
Metal disc
Fe
Deflection Arm
Metal rod
mg

1912: Victor Hess discovers cosmic rays

"The results are most easily explained by the assumption that radiation with very high penetrating power enters the atmosphere from above; the Sun can hardly be considered as the source."



What do we know 100 years later?



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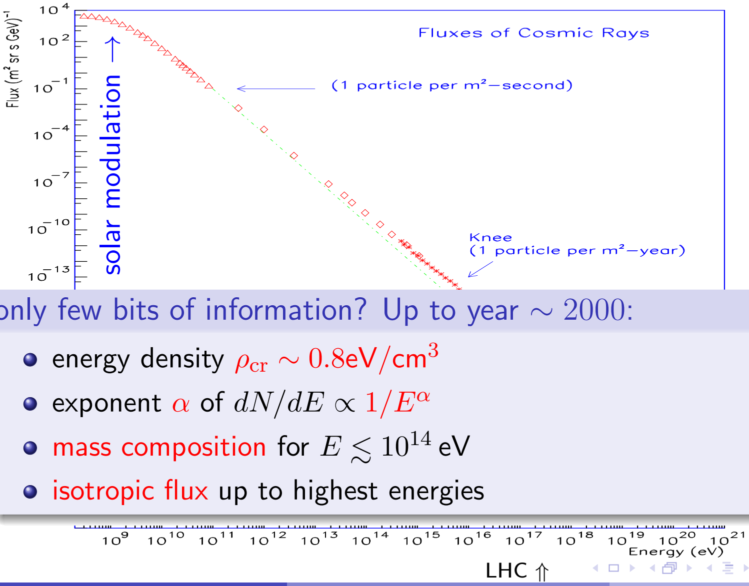
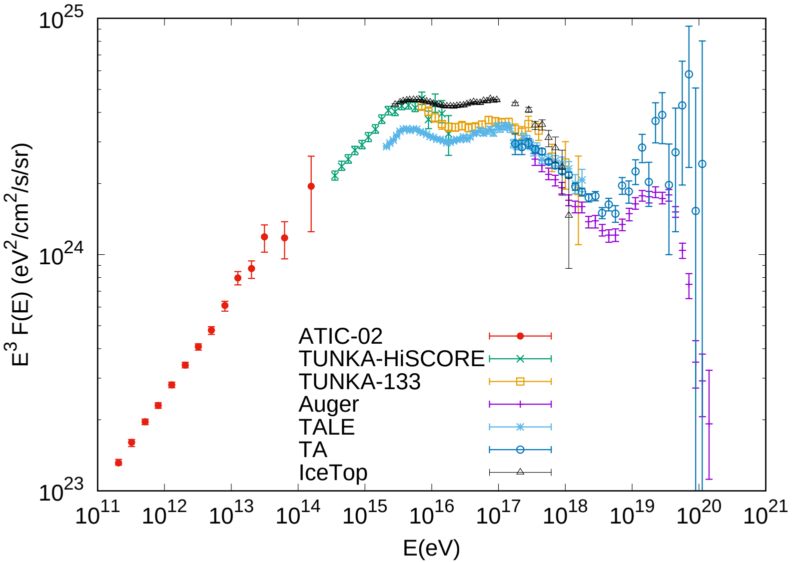


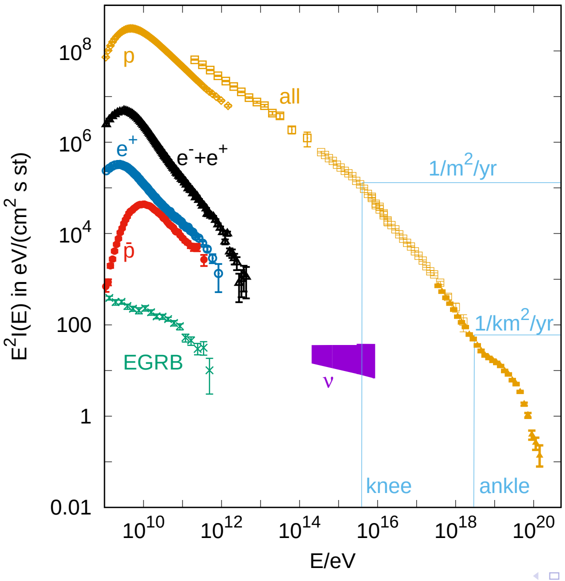
only few bits of information? Up to year ~ 2000:

- energy density $\rho_{\rm cr} \sim 0.8\text{eV}/\text{cm}^3$
- exponent α of $dN/dE \propto 1/E^\alpha$
- mass composition for $E \lesssim 10^{14}$ eV
- isotropic flux up to highest energies

Structures in CR spectrum: $E^{\alpha} I(E)$



Overall picture: $E^2 I(E)$



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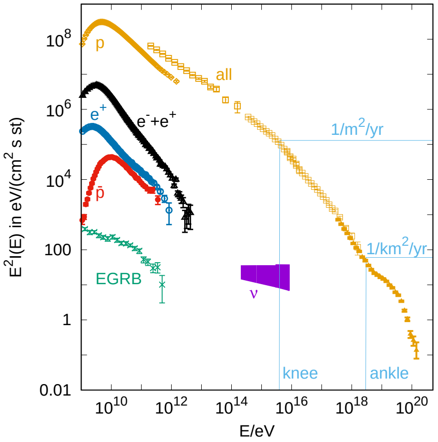
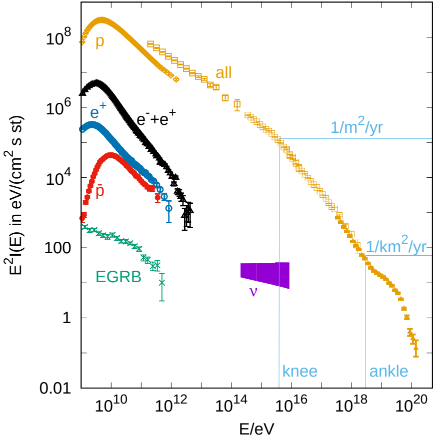


- area $\int \mathrm{dln}(E)\, E^2 I(E) \propto \int \mathrm{d}E\, E\mathrm{d}N(E)/\mathrm{d}E \propto$ energy density

Overall picture: $E^2 I(E)$

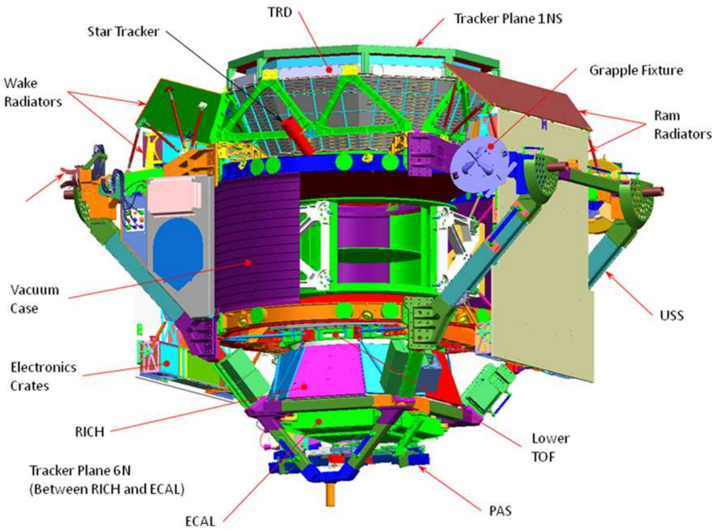


- area $\int \mathrm{d}\ln(E)\, E^2 I(E) \propto \int \mathrm{d}E\, E \mathrm{d}N(E)/\mathrm{d}E \propto$ energy density

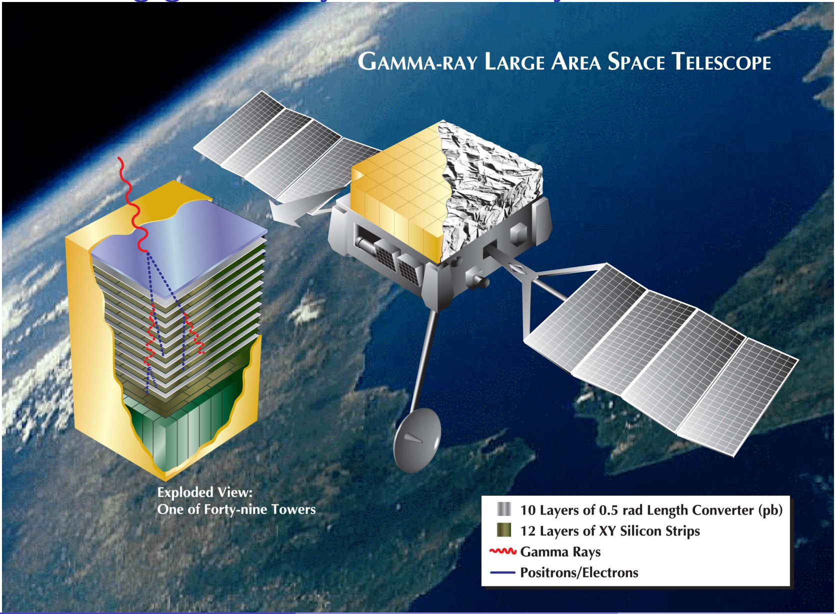
$\Rightarrow$ comparable energy in exgal. protons, neutrinos and EGBR

AMS-02

AMS-02


TRD
Star Tracker
Tracker Plane 1NS
Grapple Fixture
Wake
Radiators
Ram
Radiators
Vacuum
Case
USS
Electronics
Crates
RICH
Lower
TOF
Tracker Plane 6N
(Between RICH and ECAL)
PAS
ECAL

Observing gamma-rays or cosmic rays: GeV-TeV



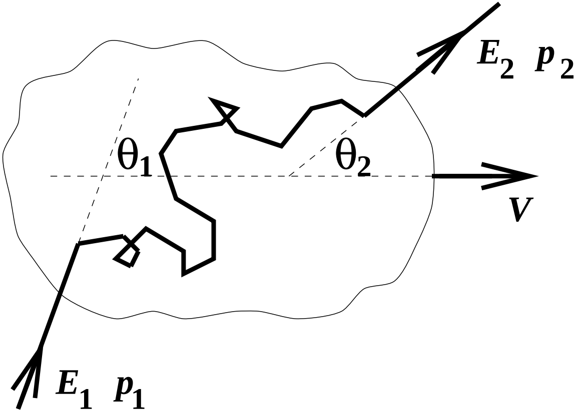
CR sources and acceleration

Plan of the lectures: CR sources and acceleration

- Fermi acceleration
- Core collaps supernovae
- Gamma-ray bursts
- Supernove remnants
  - Shock evolution
  - Diffusive shock acceleration
- Pulsars
- Active galactic nuclei

2.nd order Fermi acceleration

consider CR with initial energy E_1 “scattering” at a “cloud” moving with velocity V:



Energy gain $\xi \equiv (E_2 - E_1)/E_1$?

- Lorentz transformation 1: lab (unprimed) $\to$ cloud (primed)

$E_1' = \gamma E_1(1 - \beta \cos\vartheta_1)$ where $\beta = V/c$ and $\gamma = 1/\sqrt{1-\beta^2}$

- Lorentz transformation 2: cloud $\to$ lab

$$E_2 = \gamma E_2'(1 + \beta \cos\vartheta_2')$$

- scattering off magnetic irregularities is collisionless, the cloud is very massive

$$\Rightarrow E_2' = E_1'$$

Energy gain $\xi \equiv (E_2 - E_1)/E_1$?

$\Rightarrow E_2' = E_1'$:

- Lorentz transformation 1: lab → cloud

$$E_1' = \underbrace{\gamma E_1 (1 - \beta \cos\vartheta_1)} \quad \text{where} \quad \beta = V/c \quad \text{and} \quad \gamma = 1/\sqrt{1-\beta^2}$$

- Lorentz transformation 2: cloud → lab

$$E_2 = \gamma E_2' (1 + \beta \cos\vartheta_2')$$

$$\Rightarrow \xi = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\cos\vartheta_1 + \beta\cos\vartheta_2' - \beta^2 \cos\vartheta_1 \cos\vartheta_2'}{1-\beta^2} - 1.$$

we need average values of $\cos\vartheta_1$ and $\cos\vartheta_2'$:

Assume: CR scatters off magnetic irregularities many times in cloud
⇒ its direction is randomized,

$$\langle\cos\vartheta_2'\rangle = 0.$$

collision rate CR–cloud: proportional to their relative velocity
$(v - V\cos\vartheta_1)$:
⇒ for ultrarelativistic particles, $v = c$,

$$\frac{dn}{d\Omega_1} \propto (1 - \beta\cos\vartheta_1),$$

and we obtain

$$\langle\cos\vartheta_1\rangle = \int \cos\vartheta_1 \frac{dn}{d\Omega_1} d\Omega_1 \Big/ \int \frac{dn}{d\Omega_1} d\Omega_1 = -\frac{\beta}{3}$$

Energy gain ξ for 2.nd order Fermi:

Plugging $\langle\cos\vartheta_2'\rangle = 0$ and $\langle\cos\vartheta_1\rangle = -\frac{\beta}{3}$ into formula for ξ gives

$$\xi = \frac{1+\beta^2/3}{1-\beta^2} - 1 \simeq \frac{4}{3}\beta^2$$

since $\beta \ll 1$.

- $\xi \propto \beta^2 > 0 \Rightarrow$ energy gain
  - $O(\xi) = \beta^2$,
    because $\beta \ll 1$: average energy gain is very small
  - ξ depends on drift velocity of "clouds"

$\Rightarrow$ spectrum is not universal

Energy spectrum

- energy after n acceleration cycles

$$E_n = E_0(1+\xi)^n$$

- if escape probability per encounter is $p_{\rm esc}$, then probability to stay in acceleration region after n encounters is $(1-p_{\rm esc})^n$
- number of encounters needed to reach E_n is

$$n = \ln\left(\frac{E_n}{E_0}\right) / \ln(1+\xi)$$

- fraction of particles with energy $> E_n$ is

$$f(>E_n) = \sum_{m=n}^{\infty}(1-p_{\rm esc})^m = \frac{(1-p_{\rm esc})^n}{p_{\rm esc}}$$

Energy spectrum

- number of encounters needed to reach E is

$$n = \underbrace{\ln\left(\frac{E}{E_0}\right) / \ln\left(1+\xi\right)}$$

- fraction with energy $> E$ is

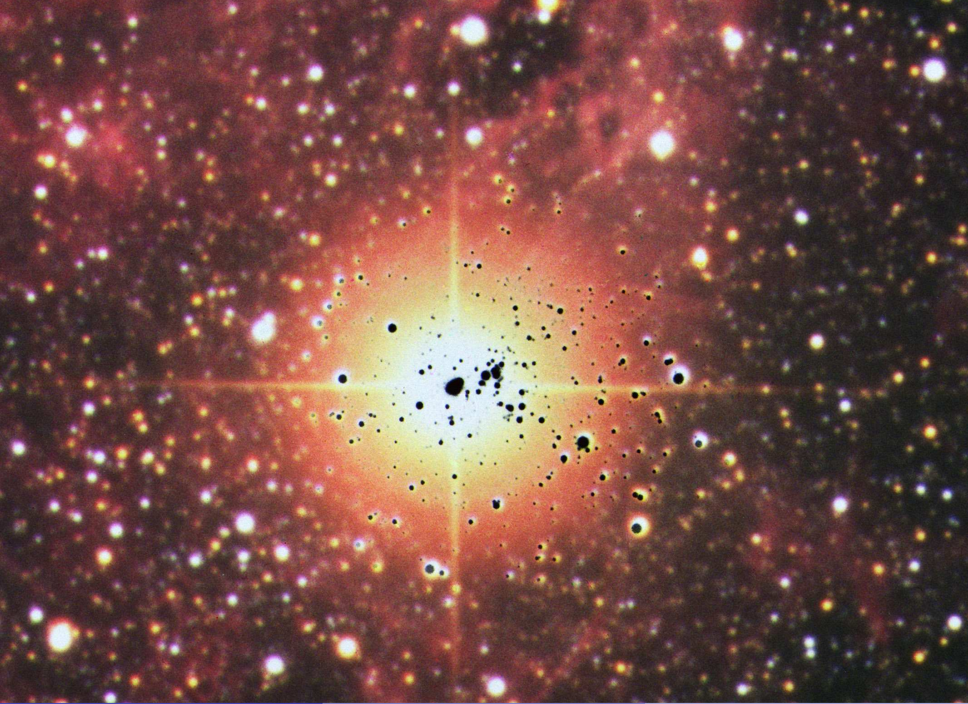
$$f(> E) = \frac{(1-p_{\rm esc})^n}{p_{\rm esc}} \propto \frac{1}{p_{\rm esc}}\left(\frac{E}{E_0}\right)^\gamma$$

where

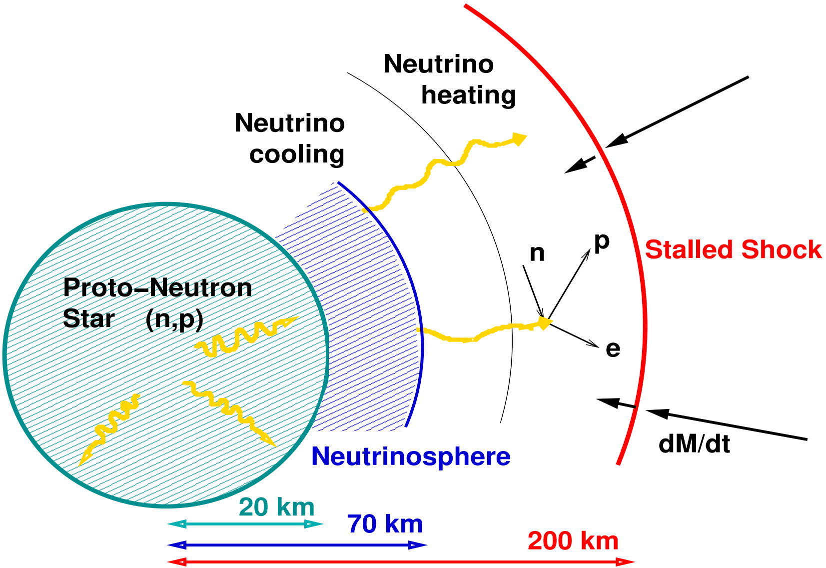
$$\gamma = \ln\left(\frac{1}{1-p_{\rm esc}}\right) / \ln(1+\xi) \approx p_{\rm esc}/\xi$$

Supernova 1987A

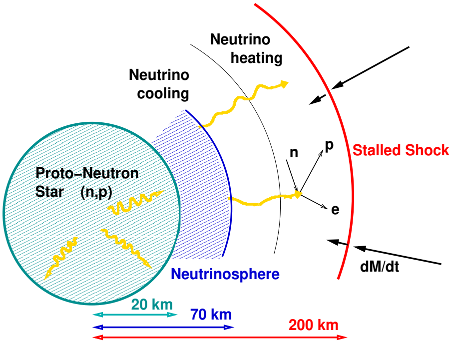
Sanduleak 69202



Neutrino driven type-II SN:



Energy budget:



Grav. binding energy
$E_b \sim 3 \times 10^{53}$ erg
$\sim 17\% M_\odot$

emitted in

- 99% neutrinos – outpowers the whole visible universe
- 1% kinetic energy
- 0.01 % photons – overshines the host galaxy

energy output:

- release of gravitational binding energy:

$$\Delta E = \left[-\frac{-GM^2}{R}\right]_{\rm star} - \left[-\frac{-GM^2}{R}\right]_{\rm NS}$$

- $R_{\rm star} \sim 10^{10}$ cm$\gg R_{NS}$ $\quad\Rightarrow$ first term dominates, although $M_{NS} < M_{\rm star}$

$$\Rightarrow \Delta E = 5 \times 10^{53} {\rm erg} \left(\frac{10{\rm km}}{R}\right) \left(\frac{M_{NS}}{1.4 M_\odot}\right)$$

energy output: where does it go?

- kinetic energy

$$\frac{1}{2}Mv^2 = 3 \times 10^{51}\text{erg}\left(\frac{M}{10M_\odot}\right)\left(\frac{v}{5000\text{km/s}}\right)$$

- optical and gravitational ($\sim \dot{Q}$) energy is even smaller

$\Rightarrow$ emitted in neutrinos, the only particles which can escape from hot core

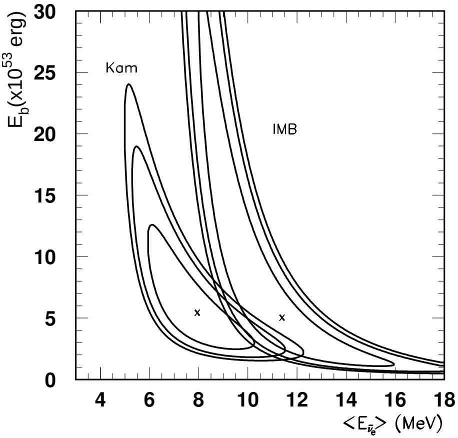
neutrino signal:

- use virial theorem for nucleon at core of proto-NS:
  $2E_{\rm kin} \sim GM_{NS}m_N/R_{NS}$
  $\Rightarrow E_{\rm kin} \sim 25$ MeV
- duration: diffusion time scale $\tau \sim R_{\rm NS}^2/\lambda$, where λ is mean free path
- $\sigma \sim G_F^2 E^2 \sim 10^{-42}$ cm$^2 (E/10\ {\rm MeV})^2$ and $\rho \sim \rho_{\rm nuc} \sim 10^{38}$ cm^{-3}

$\Rightarrow$ λ is a few meters or $O(\tau) \sim 1$s

what was learnt from SN1987A?

- confirmed principle of ν-driven SN:



fit assumes:
-thermal spectra
-equipartition

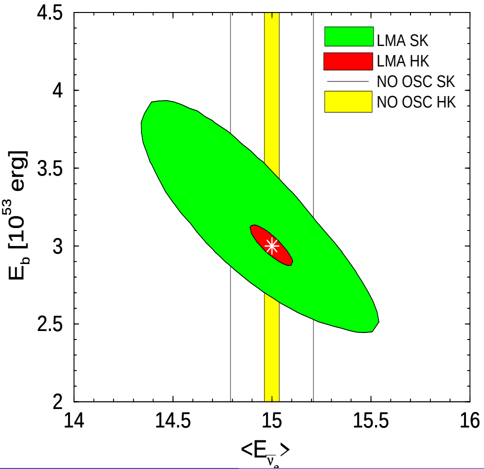
. . . and from a future SN?

- SN 1987A in LMC:
  ~ 20 Neutrinos in Kamioka and IMB detected
- future galactic SN:
  - $\sim 10^4$ Neutrinos in Super-K
  - $\sim 10^5$ Neutrinos in Hyper-K

. . . and from a future SN?

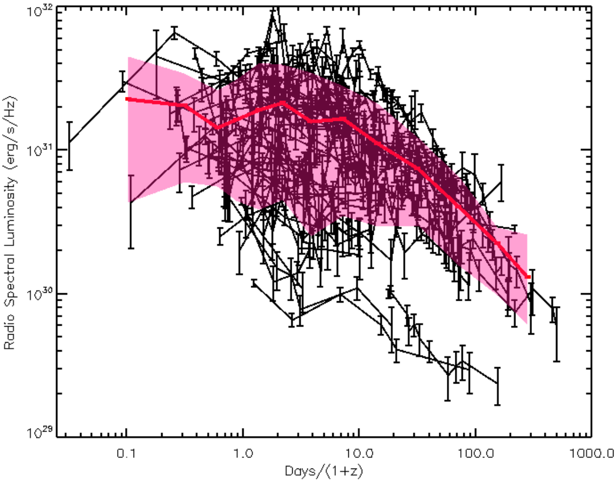
large number of events in 10–20 s allows

- study of time evolution of SN (shock)
- determination of astrophysical parameters, EoS



Gamma-ray bursts (GRBs)

- detected 1967, published in 1973
- brief flashes of gamma rays (few ms and several hours)



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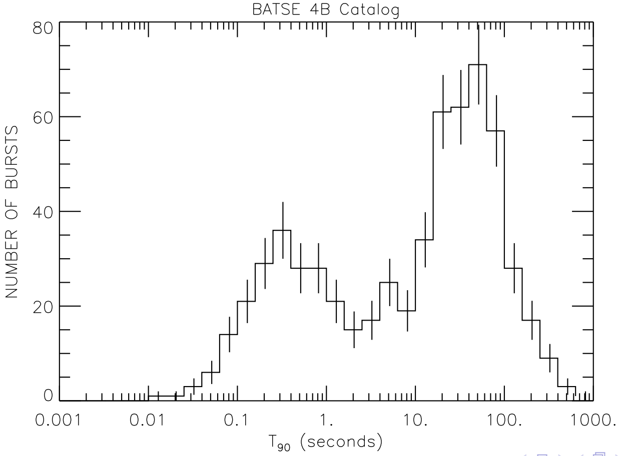

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- two classes, LGRBs and SGRBs
- assumed Galactic origin (pulsar glitches,..)
  - compactness: variability $\delta t \sim 1\,\text{ms} \Rightarrow R \lesssim c\delta t \sim 100\,\text{km}$
  - energy $E_{\rm iso} = 4\pi D^2 F = 10^{38}\text{erg}\left(\frac{D}{3\,\text{kpc}}\right)^2\left(\frac{F}{10^{-7}\text{erg/cm}^2}\right)$
  - absorption lines: 1.cyclotron line $B \sim 10^{12}$G

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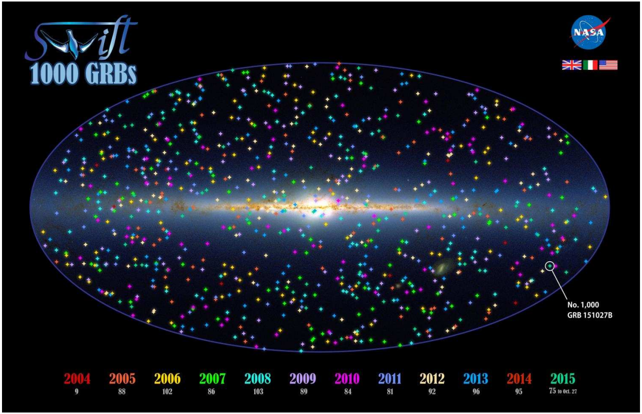
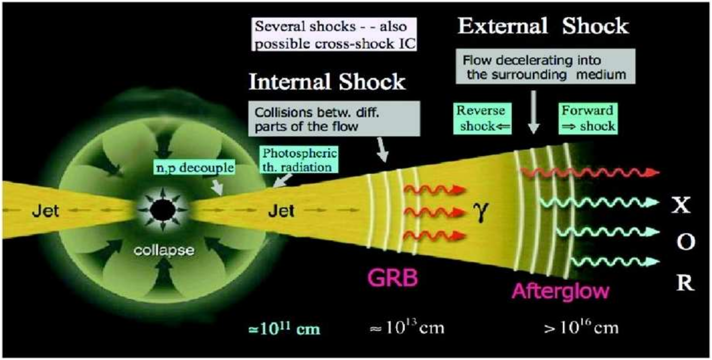
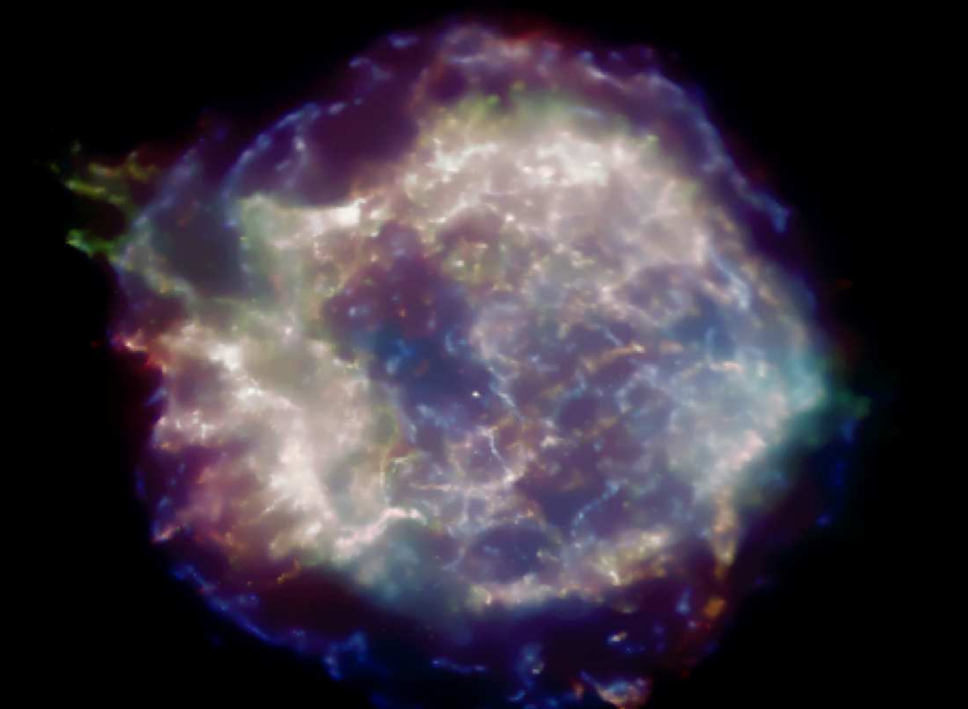


Fireball model

- cosmological distances, $E \sim 10^{51}$ erg & beaming



- $e^+e^-\gamma$ fireball collimated by funneling through surrounding matter
- NS-NS merger, failed SNe?
- LGRBs–SNeIc, SGRBs merger events



Supernova remnant

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$\Rightarrow$ problem is self-similar:

- $[E/\rho] = \frac{\mathrm{cm}^5}{\mathrm{s}^2}$, we form length

$$\lambda = \left(\frac{Et^2}{\rho}\right)^{1/5}$$

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⇒ shock radius

$$R_s(t) = \alpha\lambda(t) = \alpha\left(\frac{Et^2}{\rho}\right)^{1/5}$$

⇒ shock velocity

$$v_s(t) = \frac{\mathrm{d}R_s}{\mathrm{d}t} = \frac{2}{5}\frac{R_s}{t}$$

Supernova remnant

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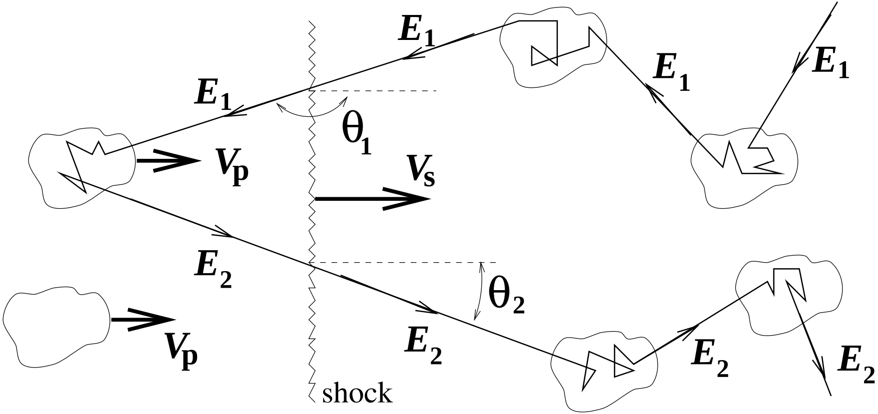
- can find α from energy conservation

Diffusive shock acceleration

consider CR with initial energy E_1 “scattering” at a shock moving with velocity V_s:



same discussion, but now different angular averages:

- projection of istropic flux on planar shock:

$$\frac{dn}{d\cos\vartheta_1} = \begin{cases} 2\cos\vartheta_1 & \cos\vartheta_1 < 0 \\ 0 & \cos\vartheta_1 > 0 \end{cases}$$

- thus $\langle\cos\vartheta_1\rangle = -\frac{2}{3}$ and $\langle\cos\vartheta_2\rangle = \frac{2}{3}$

$$\Rightarrow \xi \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

+ $\xi \propto \beta$: "efficient"
+ test particle approximation + strong shock: universal spectrum $dN/dE \propto E^{-2}$

Energy spectrum

- number of encounters needed to reach E is

$$n = \underbrace{\ln\left(\frac{E}{E_0}\right) / \ln\left(1+\xi\right)}$$

- fraction with energy $> E$ is

$$f(>E) = \frac{(1-p_{\rm esc})^n}{p_{\rm esc}} \propto \frac{1}{p_{\rm esc}} \left(\frac{E}{E_0}\right)^{\gamma} \qquad \text{where}$$

$$\gamma = \ln\left(\frac{1}{1-p_{\rm esc}}\right) / \ln(1+\xi) \approx p_{\rm esc}/\xi$$

- shock: $p_{\rm esc} \propto u_2 \Rightarrow \gamma \approx p_{\rm esc}/\xi \approx \frac{3}{u_1/u_2-1}$
- strong shock: $R \equiv u_1/u_2 = 4$ and $dN/dE \propto E^{-2}$

Fluids: Lagrangian vs. Eulerian coordinates

- consider the change of an arbitrary quantitiy $f(\boldsymbol{x},t)$ during the time $\mathrm{d}t$ as the fluid element moves from $\boldsymbol{x}$ to $\boldsymbol{x}+\mathrm{d}\boldsymbol{x}$,

$$\begin{aligned} \mathrm{d}f &= f(\boldsymbol{x}+\mathrm{d}\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t) = \\ & f(\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t)+f(\boldsymbol{x}+\mathrm{d}\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t+\mathrm{d}t)\,. \end{aligned}$$

- split the total change $\mathrm{d}f$ into a part along $\mathrm{d}t$ and a part along $\mathrm{d}\boldsymbol{x}$,

$$\mathrm{d}f(\boldsymbol{x},t)=\frac{\partial f(\boldsymbol{x},t)}{\partial t}\mathrm{d}t+\mathrm{d}\boldsymbol{x}\cdot\boldsymbol{\nabla}f(\boldsymbol{x},t+\mathrm{d}t)\,.$$

Taylor expand the second term around t, neglect the second order term $\mathrm{d}\boldsymbol{x}\mathrm{d}t$ and then divide by $\mathrm{d}t$ we obtain

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Fluids: Lagrangian vs. Eulerian coordinates

- consider the change of an arbitrary quantitiy $f(\boldsymbol{x},t)$ during the time $\mathrm{d}t$ as the fluid element moves from $\boldsymbol{x}$ to $\boldsymbol{x}+\mathrm{d}\boldsymbol{x}$,

$$\begin{aligned}\mathrm{d}f &= f(\boldsymbol{x}+\mathrm{d}\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t)= \\ & f(\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t)+f(\boldsymbol{x}+\mathrm{d}\boldsymbol{x},t+\mathrm{d}t)-f(\boldsymbol{x},t+\mathrm{d}t)\,.\end{aligned}$$

- split the total change $\mathrm{d}f$ into a part along $\mathrm{d}t$ and a part along $\mathrm{d}\boldsymbol{x}$,

$$\mathrm{d}f(\boldsymbol{x},t)=\frac{\partial f(\boldsymbol{x},t)}{\partial t}\mathrm{d}t+\mathrm{d}\boldsymbol{x}\cdot\boldsymbol{\nabla}f(\boldsymbol{x},t+\mathrm{d}t)\,.$$

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⇒ The change of the quantity f within a fixed fluid element consists of the change at a fixed coordinate, $\partial f/\partial t$, and the change due to the movement of the fluid element, $\boldsymbol{u}\cdot\boldsymbol{\nabla} f$.

⇒ introduce total (or convective) derivative with

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\,.$$

Continuity equation

- The change of the mass contained in a volume V is equal to the flow through its boundary ∂V, or in differential (Eulerian) form,

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \,.$$

- Expressing ∂t by $\mathrm{D}t$ and using $\boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = \rho \boldsymbol{\nabla} \cdot \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \rho$ gives the Lagrangian form,

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Energy equation

- replace mass density ρ by total energy density ε:

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot ((\varepsilon + P)\boldsymbol{v}) = 0 .$$

with

$$\varepsilon = \frac{nv^2}{2} + \varepsilon_{\rm int} + P + \ldots$$

sum of kinetic energy density $nv^2/2$, internal energy density $\varepsilon_{\rm int} = nU$, and pressure P.

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Momentum equation

- Straight-forward in Lagrangian form,

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = \boldsymbol{F}_{\mathrm{ex}} = -\boldsymbol{\nabla} P - \boldsymbol{\nabla}\varphi\,.$$

Thus the change $\rho \mathrm{d}v/\mathrm{d}t$ of the momentum density of a fluid element is equal to a gravitational force $\boldsymbol{F} = -\boldsymbol{\nabla}\varphi$ and a force due to a pressure gradient $-\boldsymbol{\nabla} P$. Going over to the Eulerian form,

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Ideal fluids and shocks

- ideal fluid equations plus Poisson equation,

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \,,$$

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} = \boldsymbol{F} - \boldsymbol{\nabla} P \,,$$

$$\Delta \varphi = 4\pi G \rho \,.$$

Small perturbations

- consider small, adiabatic perturbations, $\rho = \rho_0 + \rho_1$ with $\rho_0 = \text{const.}$,

$\Rightarrow$ linear, inhomogeneous wave equation

$$\partial_t^2 \rho_1 - \underbrace{c_s^2 \Delta \rho_1}_{\text{pressure}} = \underbrace{4\pi G \rho_1 \rho_0}_{\text{grav.force}}$$

for the density perturbation ρ_1 with $c_s = (\partial P / \partial \rho_1)^{1/2}$

- dispersion relation of plane waves $\exp(-\mathrm{i}(\omega t - kx))$,

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0 \,,$$

- EoS: $P = K\rho^\gamma$ with $\gamma = 5/3 \quad \Rightarrow \quad c_s = (\gamma P/\rho)^{1/2}$
- adiabatic compression with density $\rho_2 = \varepsilon \rho_1$ propagates, then $c_s \propto (\varepsilon \rho_1)^{(\gamma-1)/2}$
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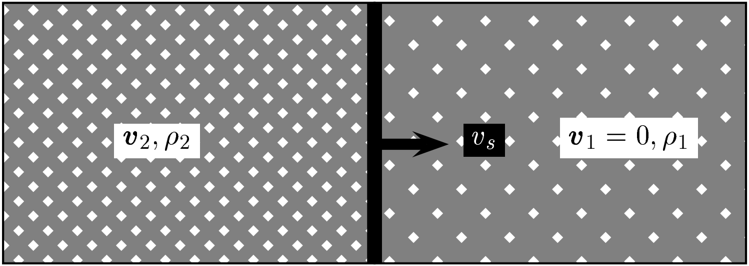
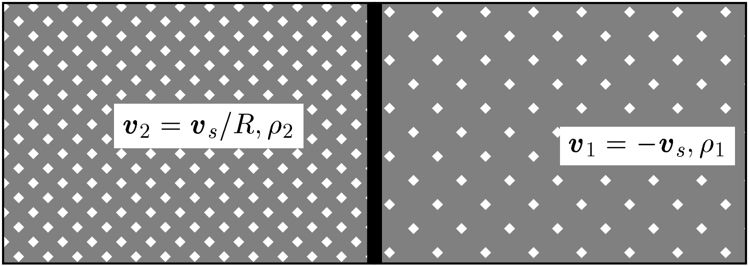
Shock in lab frame – shock rest frame





Mach number of a (strong) shock

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\rho v \left(\frac{v^2}{2} + U + \frac{P}{\rho}\right)\right] = 0$$

use again $\frac{\mathrm{d}}{\mathrm{d}x}(\rho v) = 0$

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"Rankine-Hugoniot" jump conditions

- Integrate over the discontinuity ⇒ "Rankine-Hugoniot" jump conditions

$$[\rho v]_1^2 = 0\,,$$
$$\left[P + \rho v^2\right]_1^2 = 0\,,$$
$$\left[\frac{\rho v^2}{2} + \frac{\gamma}{\gamma-1}P\right]_1^2 = 0\,,$$

- define compression ratio R as

$$R \equiv \frac{\rho_2}{\rho_1} = \frac{v_1}{v_2}$$

- Inserting first $\rho_2 = (v_1/v_2)\rho_1$ into momentum eq. gives $P_2 = P_1 + \rho_1 v_1 (v_1 - v_2)$

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- eliminate ρ_2 and P_2 from energy eq. gives quadratic equation for the ratio $t = v_2/v_1$,

$$\left(\frac{\gamma+1}{\gamma-1}\right) t^2 + \frac{2\gamma}{\gamma-1}\left(\frac{c_1^2}{v_1^2}+\gamma\right) t + \left(1+\frac{2}{\gamma-1}\frac{c_1^2}{v_1^2}\right) = 0 \qquad (3)$$

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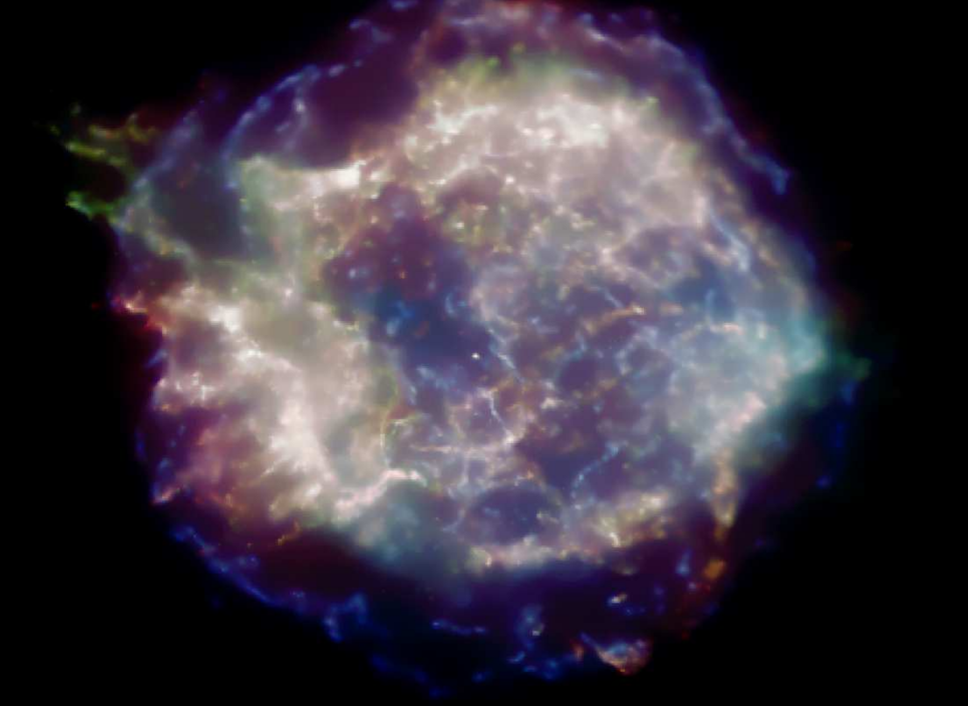
- shock frame: $v_s = v_1$; $\gamma = 5/3 \Rightarrow R = 4$,

$$v_2 = v_s/R = v_s/4$$

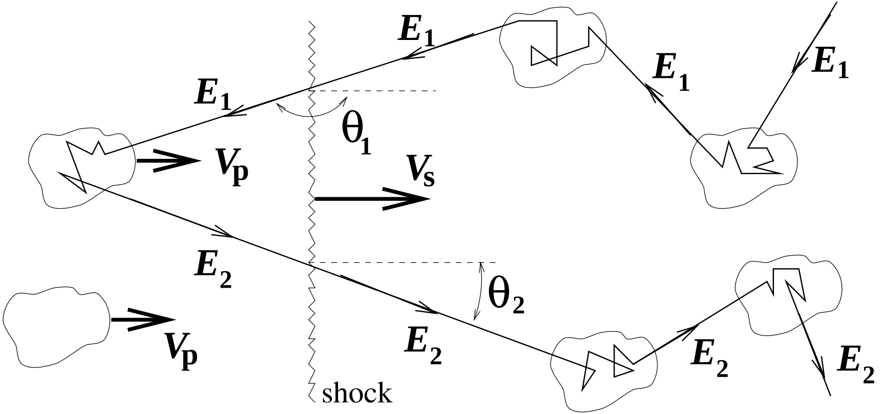
$$\rho_2 = R\rho_1 = 4\rho_1$$

$$P_2 = 3\rho_1 v_s^2/4$$



Diffusive shock acceleration

consider CR with initial energy E_1 "scattering" at a shock moving with velocity V_s:



same discussion, but now different angular averages:

- projection of istropic flux on planar shock:

$$\frac{dn}{d\cos\vartheta_1} = \begin{cases} 2\cos\vartheta_1 & \cos\vartheta_1 < 0 \\ 0 & \cos\vartheta_1 > 0 \end{cases}$$

- thus $\langle\cos\vartheta_1\rangle = -\frac{2}{3}$ and $\langle\cos\vartheta_2\rangle = \frac{2}{3}$

$$\Rightarrow \xi \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

- $p_{\rm esc} = F_{\rm esc}/F$ with $F = \pi I = cn/4$
- $F_{\rm esc} = v_2 n$

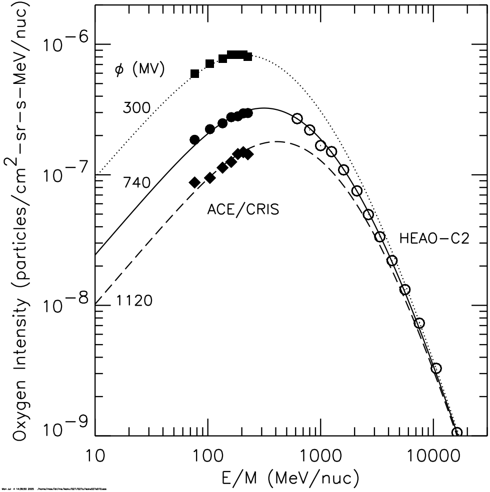
$\Rightarrow$ $p_{\rm esc} = 4v_2 n/c$

$\Rightarrow$ $\gamma \sim 2$

Plan of the lectures: Galactic CRs

- Basic observations
- Approaches
- Open questions:
  - Dipole anisotropy
  - Breaks and non-universality of primary nuclei spectra
  - Positron excess
  - Knee and the end of the Galactic CR spectrum
- Models for their explanation

Basic observations: Solar modulations



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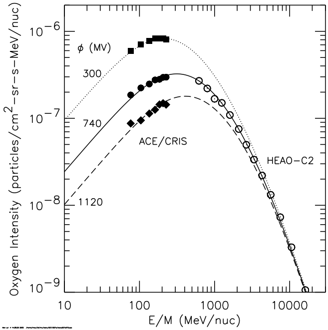
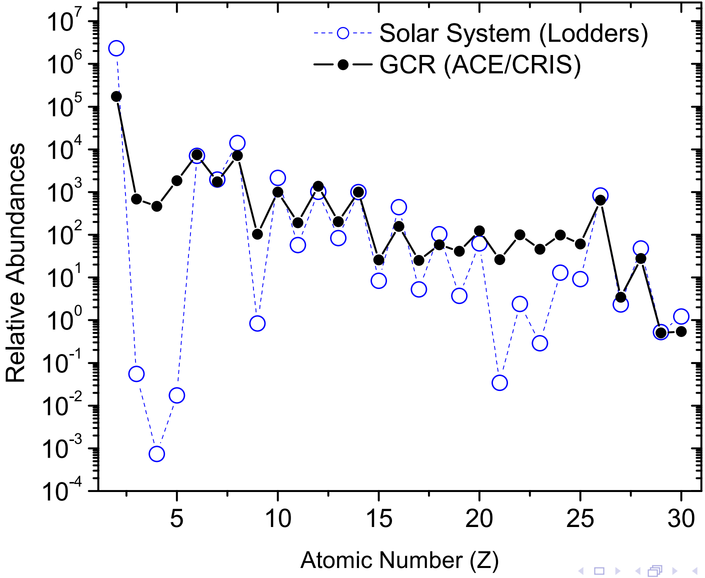


- Solar wind carries away plasma

$\Rightarrow$ solar rest frame: electric potential $\Phi_{\rm Fish}(t)$

- low-energy particles $\lesssim 20$ GV cannot penetrate Solar sytem

Basic observations: Abundances at $E/n = 5\,\text{GeV}$



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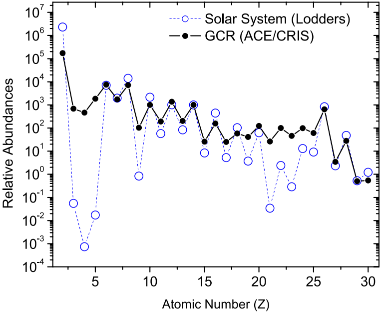

- Li, B and Ti groups strongly enriched

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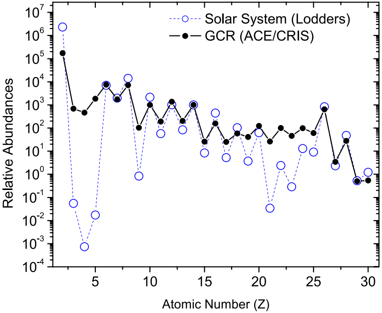
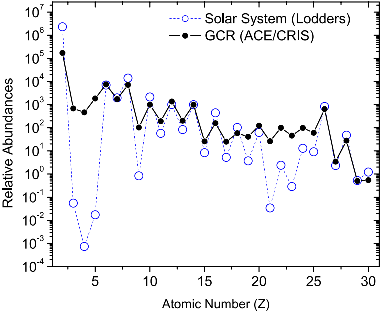


- Li, B and Ti groups strongly enriched
- spallation product of CRs on gas
- B/C fixes grammage $X \simeq 10\text{g/cm}^2 \gg X_{\text{disk}}$
- ^{9}Be/^{10}Be fixes residence time $\tau \sim 10^7\,\text{yr} \gg h/c$

Basic observations: Abundances at $E/n = 5\,\text{GeV}$



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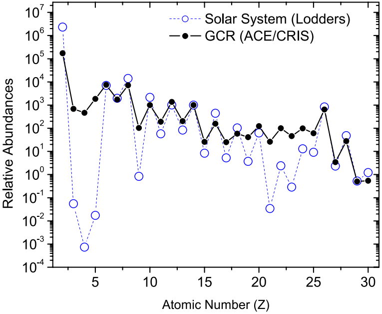

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- CRs make random walk: diffuse in Galactic magnetic field

Transport equation for CRs

$$\frac{\partial n^{(a)}}{\partial t} - \nabla_i \left[D_{ij} \nabla_j - u_i \right] n^{(a)} - \frac{\partial}{\partial p} \left[p^2 D^{(p)} \frac{\partial}{\partial p} p^{-2} n^{(a)} \right]$$

$$= -\frac{\partial}{\partial p} \left(\beta^{(a)} n^{(a)} \right)$$

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Simplest approach: Leaky box model

- Galaxy = cylinder with height 2h, escape time of CR $\tau_{\rm esc} \gg h/c$
- neglect all other effects

$$\frac{\partial n^{(a)}}{\partial t} = \frac{n^{(a)}}{\tau_{\rm esc}} = D\Delta n^{(a)} \,, \tag{6}$$

⇒ can replace the diffusion term by $n^{(a)}/\tau_{\rm esc}$

- steady-state solution, $\partial n^{(a)}/\partial t = 0$:

$$\frac{n^{(a)}(E)}{\tau_{\rm esc}} = Q^{(a)} - \left(cn_{\rm gas}\sigma^{(a)}_{\rm inel} + \Gamma^{(a)}\right) n^{(a)}(E) + cn_{\rm gas}\sum_b \int_E^\infty {\rm d}E' \, \frac{{\rm d}\sigma_{ab}(E',E)}{{\rm d}E} \, n^{(b)}(E') \,.$$

Stable secondaries

- stable secondaries like Boron $\Rightarrow \Gamma = Q = 0$:

$$\frac{n^{(a)}(E)}{\tau_{\rm esc}} = -cn_{\rm gas}(\sigma_{\rm inel}^{(a)} n^{(a)}(E) - \sum_b \sigma_{ab} n^{(b)}) \,.$$

- introduce depths $X_{\rm esc} = \beta c \rho \tau_{\rm esc}$ and $X^{(a)} = m_p/\sigma_{\rm inel}^{(a)}$
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Secondary-to-primary ratios like B/C are

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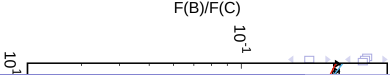

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- protons or ^{56}Fe, no decays & production can be neglected
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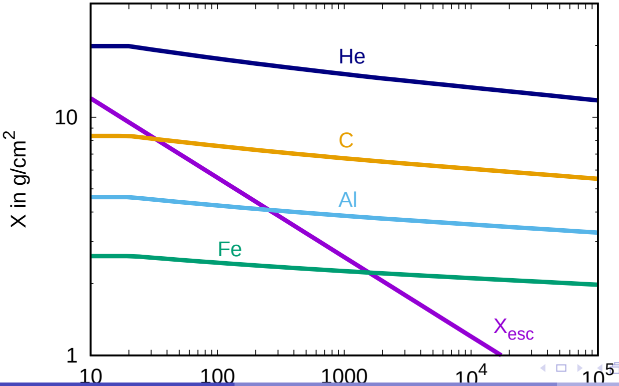

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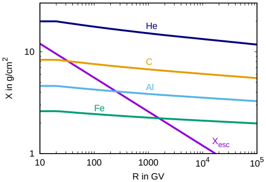
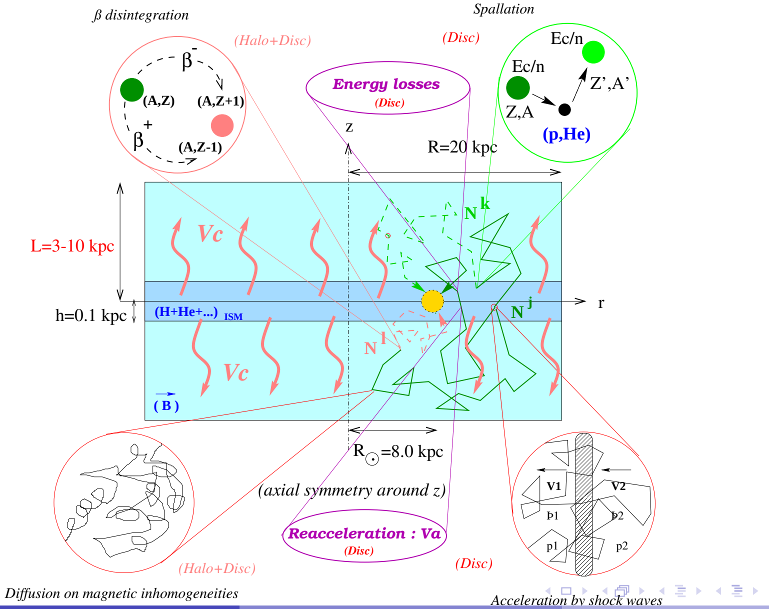


$\tau_{\rm esc} = \tau_0(\mathcal{R}/\mathcal{R}_0)^\delta$ with $\delta = 1/3$
For protons and helium, $X_{\rm p} > X_{\rm He} \gg X_{\rm esc}$ for all energies, and thus

$$n_{\rm p} = Q_{\rm p}\tau_{\rm esc} \propto Q_0 E^{-(\beta+\delta)} \,. \tag{10}$$

Standard diffusion approach:



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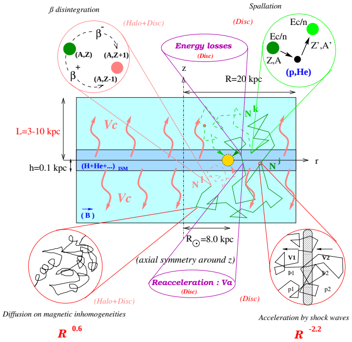

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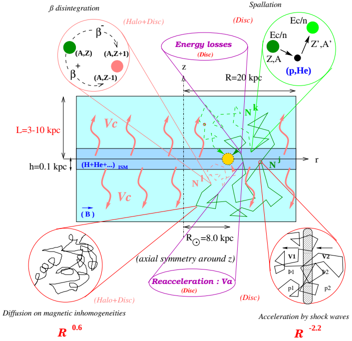

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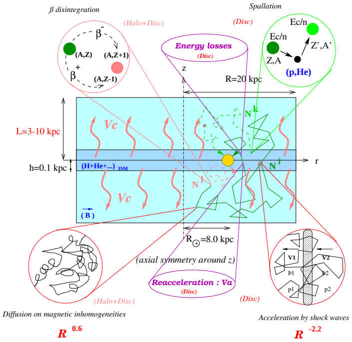

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- how important are deviations, local effects?

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  - numerical calculation straight-forward
- diffusion picture: $D(E)$ strongly degenerated with $I(E) \propto E^\alpha$ and L
- better observable: $\tau_{\rm esc}(E) = L^2/(2D) \propto 1/X$

Trajectory approach:

- use model for Galactic magnetic field: Jansson-Farrar, Psirkhov et al.,. . .
- calculate trajectories $\boldsymbol{x}(t)$ via $\boldsymbol{F}_L = q\boldsymbol{v} \times \boldsymbol{B}$.

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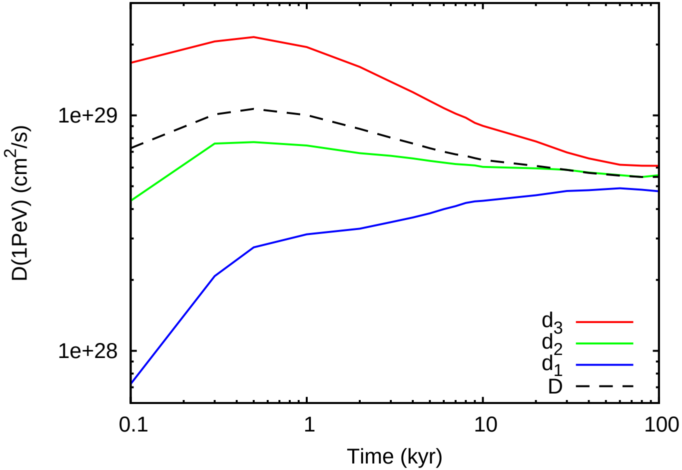
- as preparation, let's calculate diffusion tensor in pure, isotropic turbulent magnetic field

Eigenvalues of $D_{ij} = \langle x_i x_j \rangle / (2t)$ $E = 10^{15}$ eV, $B_{\rm rms} = 4\mu$G

[Giacinti, MK, Semikoz ('12)]



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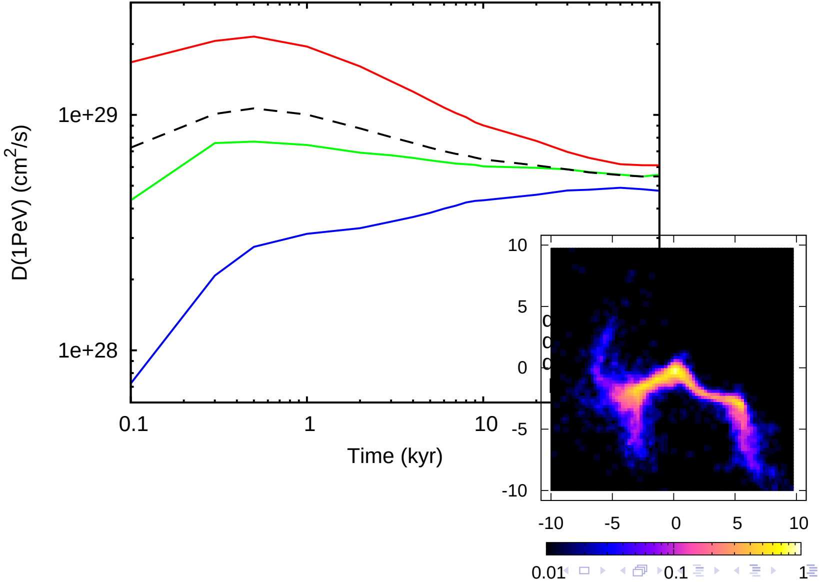

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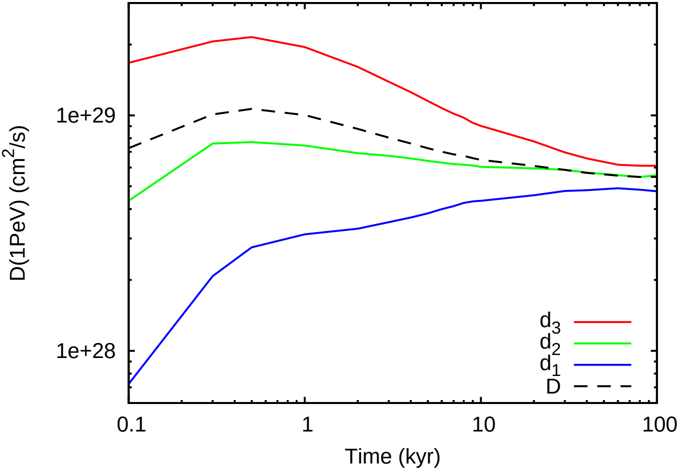

- asymptotic value is ~ 50 smaller than standard value

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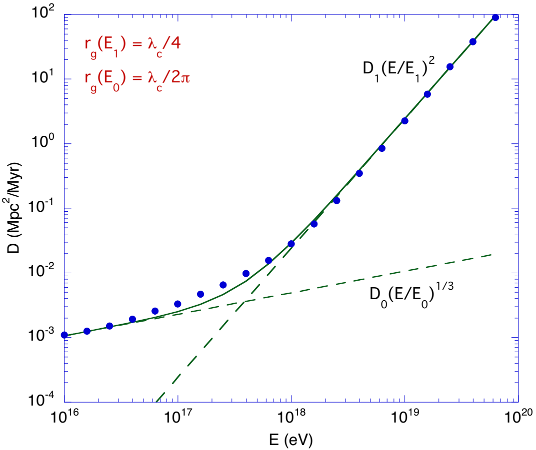

[Parizot '04]

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